

Magnon Modes for Thin Circular Vortex State Magnetic Dot

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The magnetization in a magnetic microdot made from soft magnetic materials can have a vortex-like ground state structure resulting from competition between the exchange and dipolar interactions. Normal mode magnon frequencies for such dots are calculated taking into account both exchange and magnetostatic effects. The presence of a low-lying mode as well as doublet structure with small splitting is demonstrated. Estimates of the mode frequencies for permalloy dots are obtained, and the possibility of experimental detection of such modes is discussed.

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Nanostructures fabricated from soft magnetic materials are of interest both in basic and applied magnetism with potential applications including high-density magnetic storage using an array of dot structures.^{2,3} In the following we will consider the dynamics of thin magnetic dots, with the important parameter in relation to the dot radius, R and the dot thickness, L being the exchange length, $l_0 = \sqrt{A/4\pi M_s^2}$, where A is the inhomogeneous exchange constant and M_s is the saturation magnetization. It has been shown⁴ that under certain conditions a vortex structure will be stable because of competition between the exchange and dipole interactions, and it is expected that these non-uniform states will drastically change the dynamic and static properties of a dot. For this reason dynamic properties for thin ($L \ll R$) magnetic dots in the vortex state are studied by determination of the spin wave normal mode frequencies. The method used is based on the determination of the of vortex-magnon scattering amplitude,⁵ where the non-local dipolar interaction is included by calculation of the magnetostatic energy arising from spin waves on the vortex background. A calculation of the discrete spin-wave spectrum leads to modes labeled by an azimuthal number, m , and a principal number, n . The magnon modes for the dot are compared with modes in the uniformly magnetized dot.⁶ For vortex state dots a quasi-Goldstone mode ($m = 1, n = 0$) with an anomalously small frequency appears and the doublets with $m = \pm|m|$ are split.

For consideration of the vortex state it is natural to write the magnetization, \vec{M} by use of angular variables in the polar coordinate system $M_x = M_s \sin \theta \cos \varphi$, $M_y = M_s \sin \theta \sin \varphi$, $M_z = M_s \cos \theta$. For thin enough dots with L smaller than the vortex core size the magnetization is uniform along the dot axis z , and the vortex state is given by the ansatz

$$\theta = \theta(r), \varphi = \chi \pm \varphi_0. \quad (1)$$

where r and χ are polar coordinates in the dot plane.

This distribution is typical for magnetic vortices in two-dimensional easy plane (EP) ferromagnet (FM), and the function $\theta(r)$ is determined by numerical solution of an ordinary differential equation.⁷ This gives the results that in the center of the dot ($r = 0$) $\sin \theta(r) = 0$ and for $r \gg l_0$, the value $\theta(r)$ tends to $\pi/2$ exponentially. For the magnetic dot made of soft magnetic materials the crystallographic EP anisotropy is negligible, and the demagnetization field, \vec{H}_m is dominant. Assuming that the magnetization along the dot axis z is uniform, the energy is as an integral over the dot plane,

$$W = \frac{1}{2}L \int d^2x \left[(A/M_s^2) (\nabla \vec{M})^2 - \vec{M} \vec{H}_m \right] \quad (2)$$

The sources of magnetostatic field $\vec{H}_m = -\nabla \Phi$ are both volume and surface "magnetic charges" arising from $\text{div} \vec{M}$, and from the discontinuity of the normal component of \vec{M} on the surface. For determination of the demagnetizing field the vortex distribution (1) is very simple because $\text{div} \vec{M} = 0$ and $\vec{M} \vec{r}|_{r=R}$ for $\varphi_0 = \pi/2$, thereby giving neither dot edge nor volume contributions and the sole source of field, for the ground state is M_z . For a thin enough dot this gives the contribution to the energy $-\vec{M} \vec{H}_m/2 = 2\pi M_z^2$ which is an effective EP anisotropy forming the out-of-plane vortex structure. The magnetostatic potential from the edge and volume are determined by

$$\Phi_v = \int \frac{\text{div} \vec{M}}{|\vec{r} - \vec{r}'|} d^3x, \Phi_{edge} = \int_{r=R} \frac{(\vec{M} \vec{r})}{|\vec{r} - \vec{r}'|} R d\chi dz \quad (3)$$

where $(\vec{r} - \vec{r}')^2 = r^2 + r'^2 - 2rr' \cos(\chi - \chi') + (z - z')^2$. The magnon modes are investigated using the usual procedure of expansion in small deviations from the vortex state using $\theta = \theta_0(r) + \vartheta$ and $\varphi = \chi + \pi/2 + \psi$. It is convenient to define the new small variable, $\mu = -\psi \sin \theta_0$ so that the corrections to magnetization can be expressed in the

simple form, $\delta\vec{M}/M_s = \vartheta (\vec{\chi} \cos \theta_0 - \vec{z} \sin \theta_0) + \mu \vec{r}$. The linear equations of motion for μ, ϑ can be written as

$$4\pi M_s \hat{G} \begin{pmatrix} \vartheta \\ \mu \end{pmatrix} - \frac{1}{r} \begin{pmatrix} \cos \theta_0 (\partial \Phi / \partial \chi) \\ r (\partial \Phi / \partial r) \end{pmatrix} = \frac{1}{\gamma} \frac{\partial}{\partial t} \begin{pmatrix} -\mu \\ \vartheta \end{pmatrix} \quad (4)$$

where $\Phi = \Phi_v + \Phi_{edge}$, from (3), and \hat{G} is the same dimensionless operator as for the vortex in EP magnets.⁵ Thus, in contrast to the case of EP magnets with local interactions,⁵ the equations for ϑ and μ become integro-differential.

To proceed with the solution of (4) the ansatz $\vartheta = f_m(r) \cos(\chi m + \omega t)$, $\mu = g_m(r) \sin(\chi m + \omega t)$ is used. It is easy to see that for this ansatz the values of $\nabla \vec{M}$ and $\vec{M} \vec{r}|_{r=R}$, as well as the magnetostatic potentials Φ_v and Φ_{edge} , are proportional to $\sin(\chi m + \omega t)$. Then, one obtains the set of ordinary integro-differential equation for functions $f_m(r), g_m(r)$ instead of (4) with partial derivatives.

The edge contribution to \vec{H}_m can be estimated from the integral (3), which diverges as $L \rightarrow 0$ giving a non-analytic contribution to the system energy for small L . Using asymptotic techniques, the edge contribution to the magnetostatic field can be written in the form $H_r = -\mu(R, \chi) M_s F_m(r)$, where $F_m(r) = (L/R)(r/R)^{|m|-1}$ as $r \rightarrow 0$, and reaches its maximal value independent of the small parameter L/R , with $F_m(r) \rightarrow 2\pi$, as $r \rightarrow R$ near the edge. The maximal value of the volume contribution to H_r is small (the order of L/R) over all of the dot. Thus, the main contribution to the magnetostatic energy is determined by the field H_r in the region near the edge $R - r \leq L$. The leading (logarithmic) contribution to the energy in the parameter L/R for any m is determined from the value of g on the edge only,

$$W_{edge} = 2\pi L^2 R I_m M_s^2 g^2(R) \ln \left(\frac{4R}{L} \right) \quad (5)$$

where $I_m = 2$ for $m = 0$ and $I_m = 1$ for the other modes. Therefore, to lowest order in L/R , the contribution from non-local magnetostatic potentials can be accounted by use of the effective boundary condition for $g(r)$,

$$R \frac{dg(r)}{dr} \Big|_{r=R} + g(R) \Lambda = 0, \Lambda = \frac{RL}{2\pi l_0^2} \ln \left(\frac{4R}{L} \right) \quad (6)$$

To find the frequencies of magnon modes in the lowest (logarithmic) order in L/R , one can solve (4) with $\Phi = 0$, find $\mu(r)$ and apply the boundary condition (6). The solution far from the vortex core, $r \gg l_0$ is $g_m(r) = J_m(kr) + \sigma_m(kl_0) Y_m(kr)$, where $J_m(z), Y_m(z)$ are Bessel and Neumann functions, respectively, k is a wave number, and $\sigma_m(kl_0)$ is the so-called scattering amplitude.⁵ For the case of interest $kR \leq 1$ the value of $kl_0 \leq l_0/R \ll 1$, and long-wave approximation is valid. In this case, the spin wave frequency $\omega = \omega_0 l_0 k$, where

$\omega_0 = 4\pi\gamma M_s$, and the asymptotics for $\sigma_m(kl_0)$ found in⁵ can be used. For estimates, we will use the values for permalloy, $A^{1/2}/M_s = 17$ nm, that gives $l_0 = 4.8$ nm and the characteristic demagnetizing field $4\pi M_s = 1.012$ T, leading to the following results.

In the first approximation on $kl_0 \ll 1$ the scattering amplitude is small and $g_m(r) \cong J_m(kr)$. The frequencies of modes with $m = |m|$ and $m = -|m|$ are equal and are determined by $\omega_{n,m} = \varpi_{n,|m|} = 4\pi\gamma M_s (\kappa_{n,m} l_0 / R)$, where $\kappa_{n,m}$ is the n -th solution of $\kappa J'_m(\kappa) + \Lambda J_m(\kappa) = 0$. The value of $\kappa_{n,m}$ falls between the n -th root of the Bessel function $j_{m,n}$ and the n -th root of its derivative $j'_{m,n}$, $\kappa \rightarrow j_{m,n}$ at $\Lambda \gg m$ and $\kappa \rightarrow j'_{m,n}$ in the opposite case. Taking into account σ_m , the value of which depends on the sign of m ,⁵ produces the splitting of these doublets for the modes with $m \neq 0$. The scattering amplitude is maximal (linear in k) for the modes with $m \pm 1$, with $\sigma_m = \pi k l_0 m / 4 |m|$,⁵ and the splitting of this doublet is the order of the next power on the small parameter l_0/R , $\omega_{m=-1} - \omega_{m=1} \approx 6.16 \omega_0 (l_0/R)^2$. Thus, for a typical R like 100 nm, this splitting is of order $(0.5 \div 2)$ GHz and is much smaller than the mean frequency, see Fig.1.

Large values of the scattering amplitude leads to the most interesting feature of the vortex state dot frequencies, namely, the appearance of a low lying mode with $m = 1$ and $n = 0$ corresponding to vortex displacement from the center of the dot, the so-called translational Goldstone mode (TGM), for which $J_1(kR) \sim (kl_0) Y_m(kR)$ and $kR \ll 1$.⁵ For vortex state magnetic dots the frequency of the TGM mode, shown in Fig.2, depends non-monotonically on R , $\omega_{TGM} = \omega_0 (l_0/R)^2 (\Lambda - 1)/(\Lambda + 1)$. Comparison of Figs. 1 and 2 shows that it has the same order of magnitude as the doublet splitting for higher modes with $|m| = 1$, and it much smaller than ϖ_1 or smaller than frequencies for homogeneously magnetized dots. The frequency of TGM found from the effective equation of motion for the coordinate of the vortex \vec{X} with some model assumptions about static vortex energy shows different dependence on the dot radius,^{8,9} namely, $\omega \propto 1/R^3$ in Ref. 8 and $\omega \propto 1/R$ in Ref. 9. In contrast with,^{8,9} we do not need in any model assumption and our approach is based on the analysis of Eq.(4).

The modes with $m = 0$ are singlets having cylindrical symmetry, and their frequencies for $n = 0$ and $n = 1$ are shown in Fig. 1. The lower mode with $m = 0, n = 0$ can be considered as coupled oscillations of the vortex core size and φ_0 defined in (1). It can be observed in resonance experiments with an ac field perpendicular to the dot plane.

Modes with $|m| > 1$ have small values of $\sigma_m \propto (kl_0)^\eta$, $\eta \geq 4$ for different m with small doublet splitting, $(\omega_{n,-m} - \omega_{n,m}) / \varpi_{n,|m|} \approx (l_0/R)^\eta$, that cannot be seen on the scale used for Fig. 1. The mean frequencies $\varpi_{n,|m|}$ of these modes for the vortex state dots have the same order of magnitude as for the homogeneous states modes calculated in.⁶

In summary, it is shown that the dynamics of the magnetic dot in the vortex state differs significantly from

the case of uniformly magnetized dots. The magnon mode corresponding to oscillations of the vortex position (TGM, $m = 1, n = 0$) have the lowest frequency. The next mode ($m = 0, n = 0$) describes the oscillations of the vortex core size. These two modes can potentially be detected in resonance experiments, with ac - field parallel and perpendicular to the dots plane, respectively. Higher modes with $m \neq 0$ forms doublets with maximal splitting less than 1 GHz.

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FIG. 1: Mode frequencies as a function of dot radius R .

FIG. 2: Frequency of the TGM mode versus dot radius R for different values of dot thickness L .